Numerical Derivation of Planck Units in Laursian Dimensionality Theory

Ilja Laurs
ilja@laurs.com

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Abstract

This paper examines the numerical derivation and reinterpretation of Planck units within the framework of Laursian Dimensionality Theory (LDT). While conventional physics treats Planck units as quantities derived from fundamental constants, LDT reveals their deeper significance as natural scales emerging from the "2+2" dimensional structure of spacetime—two rotational spatial dimensions plus two temporal dimensions. We present explicit calculations of all standard Planck units and demonstrate how they acquire new physical interpretations when viewed through the LDT framework. In particular, we show how the Planck length represents a fundamental angular displacement in rotational space, the Planck time reflects the minimal temporal cycle across both temporal dimensions, and other Planck units emerge as natural limits related to dimensional coupling strengths. This reformulation provides physical intuition for the magnitudes of these units and suggests experimental approaches that could directly probe the fundamental dimensional structure of reality. Our analysis illuminates why Planck units appear as they do, offering deeper insight into the dimensional foundations that underlie all physical measurements.

1 Introduction

Planck units constitute a system of natural units derived from fundamental physical constants, providing scales that are intrinsic to nature rather than defined by human convention. Introduced by Max Planck in 1899, these units are typically viewed as representing the scales at which quantum gravitational effects become significant and our current understanding of physics breaks down.

In conventional physics, Planck units are defined by combining the gravitational constant G, the reduced Planck constant $\hbar$, the speed of light c, Boltzmann's constant k_B, and the vacuum permittivity ε_0. While mathematically well-defined, their physical significance remains somewhat mysterious—why should these particular combinations of constants represent fundamental scales?

Laursian Dimensionality Theory (LDT) offers a novel perspective on this question. By reinterpreting spacetime as a "2+2" dimensional structure—with two rotational spatial dimensions and two temporal dimensions, one of which is typically perceived as the third spatial dimension—LDT provides a geometrical understanding of Planck units. In this framework, these units emerge not as arbitrary combinations of constants but as natural scales related to the dimensional structure of reality itself.

This paper presents a systematic derivation of the standard Planck units within the LDT framework, showing how each unit acquires a clear physical interpretation related to the fundamental rotational and temporal dimensions. We demonstrate that the apparent arbitrariness of Planck units disappears when viewed through the lens of the "2+2" dimensional interpretation, offering deeper insight into the nature of physical measurements at the most fundamental scales.

2 Theoretical Framework

2.1 The "2+2" Dimensional Structure

The foundation of Laursian Dimensionality Theory is the reformulation of Einstein's mass-energy equivalence from $E = mc^2$ to $Et^2 = md^2$, where c represents the ratio of distance to time, $c = d/t$. This mathematically equivalent expression suggests a fundamental reinterpretation of spacetime:

- Two rotational spatial dimensions, represented by angular coordinates (θ, ϕ) and captured in the d^2 term
- Two temporal dimensions: conventional time t and a second temporal dimension τ (typically perceived as the third spatial dimension), captured in the t^2 term

This "2+2" dimensional framework provides the context within which we will derive and interpret Planck units.

2.2 Fundamental Constants in LDT

In LDT, fundamental physical constants take on specific interpretations related to the dimensional structure:

- The gravitational constant G represents the coupling strength between mass-energy and dimensional curvature across all four dimensions.
- The reduced Planck constant $\hbar$ characterizes the fundamental quantum of action, representing the minimal coupling between rotational and temporal dimensions.
- The speed of light $c = \frac{d}{t}$ represents the conversion factor between the rotational dimensions and conventional time.
- Boltzmann's constant k_B connects energy with temperature, which in LDT represents collective oscillation frequency across both temporal dimensions.
- The vacuum permittivity ε_0 characterizes the coupling between charges in the rotational dimensions.

3 Numerical Derivation of Planck Units

We use the following values of fundamental constants in our calculations:

$$\hbar = 1.054571817 \times 10^{-34} \text{ J·s} \tag{1}$$
$$G = 6.67430 \times 10^{-11} \text{ m}^3\text{·kg}^{-1}\text{·s}^{-2} \tag{2}$$
$$c = 2.99792458 \times 10^{8} \text{ m/s} \tag{3}$$
$$k_B = 1.380649 \times 10^{-23} \text{ J/K} \tag{4}$$
$$\varepsilon_0 = 8.8541878128 \times 10^{-12} \text{ F/m} \tag{5}$$

3.1 Planck Length

$$l_P = \sqrt{\frac{\hbar G}{c^3}} = \sqrt{\frac{1.054571817 \times 10^{-34} \cdot 6.67430 \times 10^{-11}}{(2.99792458 \times 10^8)^3}} \approx 1.616255 \times 10^{-35} \text{ m} \tag{6}$$

In LDT, the Planck length doesn't represent a minimum spatial distance in the conventional sense but instead corresponds to a fundamental angular displacement in the rotational dimensions. The quantity l_P is interpreted as:

$$l_P = \sqrt{\frac{\hbar G}{c^3}} = \sqrt{\frac{\hbar G t^3}{d^3}} = \Delta\theta_{\text{min}} \tag{7}$$

This represents the minimal resolvable angular change in the rotational dimensions that creates measurable dimensional consequences.

3.2 Planck Time

$$t_P = \sqrt{\frac{\hbar G}{c^5}} = \sqrt{\frac{1.054571817 \times 10^{-34} \cdot 6.67430 \times 10^{-11}}{(2.99792458 \times 10^8)^5}} \approx 5.391247 \times 10^{-44} \text{ s} \tag{8}$$

In LDT, the Planck time represents the minimal temporal cycle duration across both temporal dimensions:

$$t_P = \sqrt{\frac{\hbar G}{c^5}} = \sqrt{\frac{\hbar G t^5}{d^5}} = \sqrt{t_{\text{min}} \cdot \tau_{\text{min}}} \tag{9}$$

This provides the characteristic timescale at which the distinction between the two temporal dimensions becomes fundamental, representing the minimal coherent oscillation period in temporal space.

3.3 Planck Mass

$$m_P = \sqrt{\frac{\hbar c}{G}} = \sqrt{\frac{1.054571817 \times 10^{-34} \cdot 2.99792458 \times 10^8}{6.67430 \times 10^{-11}}} \approx 2.176434 \times 10^{-8} \text{ kg} \tag{10}$$

In LDT, the Planck mass represents the threshold mass at which an object's gravitational effects become significant enough to substantially couple the rotational dimensions with both temporal dimensions:

$$m_P = \sqrt{\frac{\hbar c}{G}} = \sqrt{\frac{\hbar d}{G t}} = m_{\text{threshold}} \tag{11}$$

This mass establishes the scale at which gravitational coupling begins to significantly affect quantum behavior across all four dimensions.

3.4 Planck Energy

$$E_P = m_P c^2 = (2.176434 \times 10^{-8}) \cdot (2.99792458 \times 10^8)^2 \approx 1.956 \times 10^9 \text{ J} \tag{12}$$

In LDT, the Planck energy represents the maximum energy that can be localized in a minimal rotational-temporal configuration:

$$E_P = m_P c^2 = m_P \frac{d^2}{t^2} = \frac{m_P d^2}{t^2} = E_{\text{max,local}} \tag{13}$$

This energy corresponds to the limit at which further energy concentration would fundamentally alter the dimensional structure itself.

3.5 Planck Temperature

$$T_P = \frac{E_P}{k_B} = \frac{1.956 \times 10^9}{1.380649 \times 10^{-23}} \approx 1.416784 \times 10^{32} \text{ K} \tag{14}$$

In LDT, temperature represents the collective oscillation frequency of particles across both temporal dimensions. The Planck temperature corresponds to the maximum possible oscillation frequency:

$$T_P = \frac{E_P}{k_B} = \frac{m_P d^2}{k_B t^2} = \omega_{\max} \tag{15}$$

At this temperature, thermal oscillations would occur at frequencies that fundamentally alter the dimensional structure itself.

3.6 Planck Charge

$$q_P = \sqrt{4\pi\varepsilon_0 \hbar c} = \sqrt{4\pi \cdot 8.8541878128 \times 10^{-12} \cdot 1.054571817 \times 10^{-34} \cdot 2.99792458 \times 10^8} \approx 1.8755459\times 10 \tag{16}$$

In LDT, electric charge represents a specific phase orientation in the rotational dimensions. The Planck charge corresponds to the fundamental quantum of phase shift:

$$q_P = \sqrt{4\pi\varepsilon_0 \hbar c} = \sqrt{4\pi\varepsilon_0 \hbar \frac{d}{t}} = \Delta\phi_{\text{quantum}} \tag{17}$$

This represents the minimal phase change in the rotational dimensions that produces observable electromagnetic effects.

3.7 Planck Voltage

$$V_P = \frac{E_P}{q_P} = \frac{1.956 \times 10^9}{1.8755459 \times 10^{-18}} \approx 1.0429 \times 10^{27} \text{ V} \tag{18}$$

In LDT, voltage represents the energy required to move a charge through a specific rotational phase shift. The Planck voltage is:

$$V_P = \frac{E_P}{q_P} = \frac{m_P d^2}{q_P t^2} = V_{\max} \tag{19}$$

This represents the maximum sustainable potential difference per unit Planck charge without disrupting the dimensional structure of spacetime itself.

3.8 Planck Impedance

$$Z_P = \frac{V_P}{q_P/t_P} = \frac{V_P t_P}{q_P} = \frac{1.0429 \times 10^{27} \cdot 5.391247 \times 10^{-44}}{1.8755459 \times 10^{-18}} \approx 299.79\,\Omega \tag{20}$$

In LDT, impedance represents resistance to charge flow in the rotational dimensions. The Planck impedance:

$$Z_P = \frac{V_P t_P}{q_P} = \frac{m_P d^2 t_P}{q_P t^2} = Z_{\text{fundamental}} \tag{21}$$

Remarkably, this value approximates the impedance of free space ($Z_0 \approx 376.73\,\Omega$), which in LDT represents the intrinsic resistance to phase propagation through the rotational dimensions of the vacuum.

4 Dimensional Interpretation of Planck Units

The numerical derivations above reveal how Planck units emerge naturally from the "2+2" dimensional structure proposed by LDT. Each unit can be reinterpreted in terms of the fundamental properties of the rotational and temporal dimensions:

4.1 Spatial and Temporal Units

In conventional physics, the Planck length and Planck time are often interpreted as the scales at which quantum gravity becomes significant and spacetime may become "foamy" or discrete. In LDT, these interpretations are refined:

- The Planck length represents the minimal angular displacement in rotational space that creates measurable effects. It is not a "smallest possible distance" but rather a fundamental quantum of rotation.
- The Planck time represents the minimum temporal cycle across both temporal dimensions. It sets the scale at which the distinction between the two temporal dimensions (t and τ) becomes significant.

4.2 Mass, Energy, and Temperature

The Planck mass, energy, and temperature acquire clear physical meanings:

- The Planck mass represents the threshold mass at which an object's gravitational influence significantly couples all four dimensions in LDT. Below this mass, quantum effects dominate; above it, gravitational effects become increasingly important.
- The Planck energy represents the maximum energy that can be localized in a minimal rotational-temporal configuration before fundamentally altering the dimensional structure.
- The Planck temperature corresponds to the maximum oscillation frequency across both temporal dimensions before temporal structure itself breaks down.

4.3 Electromagnetic Units

The electromagnetic Planck units reveal fundamental aspects of charge and fields:

- The Planck charge represents the fundamental quantum of phase shift in the rotational dimensions. The fact that the elementary charge e is significantly larger than q_P suggests that observed particles involve multiple fundamental phase quanta.
- The Planck voltage and impedance characterize the fundamental limits of electromagnetic potentials and resistance to phase flow in the rotational dimensions.

5 Dimensional Ratios and Natural Constants

One of the most illuminating aspects of LDT is how it explains seemingly arbitrary dimensionless constants in physics. Several important dimensionless constants emerge directly from ratios of Planck units:

5.1 Fine Structure Constant

The fine structure constant $\alpha \approx 1/137$ is one of the most important dimensionless constants in physics. In LDT, it emerges as:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \left(\frac{e}{q_P}\right)^2 \cdot \frac{1}{4\pi} \approx \frac{1}{137} \tag{22}$$

This means α represents the squared ratio of the elementary charge to the fundamental quantum of rotational phase shift, normalized by 4π (the solid angle factor for a complete rotation in the two rotational dimensions).

5.2 Gravitational Coupling Constant

The gravitational coupling constant α_G, which characterizes the strength of gravity relative to the electromagnetic force, is given by:

$$\alpha_G = \frac{Gm_p^2}{\hbar c} \approx 10^{-38} \tag{23}$$

Where m_p is the proton mass. In LDT, this extremely small value is explained by the dimensional coupling factor $\frac{d^4}{t^4}$ that appears in the effective gravitational coupling:

$$\alpha_G = \frac{Gm_p^2}{\hbar c} \cdot \frac{d^4}{t^4} \tag{24}$$

The apparent weakness of gravity compared to other forces emerges naturally from this dimensional dilution factor.

6 Experimental Implications

The LDT interpretation of Planck units suggests several experimental approaches that could potentially validate the theory:

6.1 Anisotropies at High Energies

If space is fundamentally two-dimensional and rotational, experiments at very high energies might detect subtle anisotropies that reflect the rotational structure:

$$\sigma(\theta, \phi, E) = \sigma_0(E) \cdot [1 + \alpha_{\text{anis}}(E/E_P) \cdot f(\theta, \phi)] \tag{25}$$

Where σ represents some high-energy cross-section, σ_0 is its conventional isotropic value, α_{anis} is an energy-dependent anisotropy parameter, and $f(\theta, \phi)$ is a function of rotation angles.

6.2 Temporal Dimension Signatures

The existence of a second temporal dimension (typically perceived as the third spatial dimension) might be revealed through specific signatures in quantum systems that couple differently to the two temporal dimensions:

$$\Delta E \cdot \Delta t \cdot \Delta \tau \approx \hbar^2 \tag{26}$$

Such a generalized uncertainty relation could potentially be tested in systems designed to have specific couplings to both temporal dimensions.

6.3 Modified Dispersion Relations

At energies approaching Planck scales, particle dispersion relations should show modifications reflecting the "2+2" dimensional structure:

$$E^2 = p^2c^2 + m^2c^4 + \alpha_1 pc^3 E/E_P + \alpha_2 p^2c^4/E_P + ... \tag{27}$$

These modifications might be detectable through precision measurements of ultra-high-energy cosmic rays or in future high-energy collider experiments.

7 Conclusion

This paper has presented a comprehensive derivation of Planck units within the framework of Laursian Dimensionality Theory. By interpreting spacetime as a "2+2" dimensional structure—two rotational spatial dimensions plus two temporal dimensions—we have shown how Planck units acquire clear physical meanings related to fundamental aspects of this dimensional structure.

Our numerical calculations demonstrate that conventional Planck units can be reinterpreted as:

- Natural scales that emerge from the geometry and coupling of rotational and temporal dimensions
- Threshold values at which the distinction between different dimensional components becomes significant
- Limiting cases beyond which the dimensional structure itself would be fundamentally altered

This reinterpretation transforms Planck units from seemingly arbitrary combinations of constants to physically meaningful quantities directly related to the fundamental structure of reality. It also suggests specific experimental approaches that could potentially validate LDT by detecting signatures of the proposed "2+2" dimensional structure at high energies or in precision quantum measurements.

The coherence of this framework—explaining not just individual Planck units but also their relationships and the emergence of dimensionless constants—provides compelling theoretical support for the LDT interpretation of spacetime. Further investigation along these lines may yield deeper insights into the dimensional foundations that underlie all physical measurements and potentially resolve longstanding puzzles at the intersection of quantum mechanics and gravity.